Math 241: Midterm 1

Name: NetID:

Circle your discussion section:

Professor Anema:

- ADA: 8am Field
- ADB: 9am Wen
- ADC: 10am Livesay
- ADD: 11am Livesay
- ADE: Noon Golze
- AD1: 11am Klajbor Goderich
- ADF: 1pm Golze
- AD2: 1pm Donepudi
- ADG: 2pm Shinkle
- ADH: 3pm Shinkle
- ADI: 4pm Field
- ADK: 9am Zhang
- ADL: 10am Zhang
- ADM: 2pm Li
- ADN: 3pm Li

Professor Bell:

- BDA: 8am Dunn
- BDB: 9am Dunn
- BDC: 10am Butler
- BDD: 11am Butler
- BDE: Noon Kaplan
- BDF: 1pm Ahmed
- BDG: 2pm Wen
- BDH: 3pm Tatum
- BDI: 4pm Tatum
- BDJ: 9am Roman-Garcia
- BDK: 10am Roman-Garcia
- BDL: Noon Okano
- BDM: 2pm Carmody
- BDN: 3pm Shin
- BDO: 4pm Okano
- BDR: Noon Carmody
- BDS: 10am Shin

Instructions: You have **75 minutes** to complete this exam. There are **45 points** available and not all problems are weighted equally. Calculators, books, notes, and suchlike aids are **not** permitted. **When space is provided, show work that justifies your answer** as in those problems **credit will not be given** for correct answers without proper justification. Work written outside of the space provided for a problem will **not** be graded.

Do not open exam until instructed.

Do not write in the space below or in the similar areas on each page of the exam. These are reserved for grading.

1. (a) Compute the dot product of $\langle 1, -3, -2 \rangle$ and $\langle 2, 1, 3 \rangle$. **(1 point)**

Answer:

(b) Compute $\langle 2, 1, 3 \rangle \cdot \langle 3, 1, 0 \rangle \times \langle 1, -1, 2 \rangle$. **(2 points)**

Answer:

(c) Find the area of the triangle whose vertices are $(2, 1, 3)$, $(3, 1, 0)$ and $(1, -1, 2)$. **(2 points)**

Answer:

2. **(1 point each)** Which of the following properties hold for all vectors $\mathbf{u}$ and $\mathbf{v}$ and scalars c and d? For each property, circle either True or False.

   (a) $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ True / False

   (b) $\mathbf{u}\times\mathbf{v}=\mathbf{v}\times\mathbf{u}$ True / False

   (c) $\mathbf{u}+\mathbf{v}=\mathbf{u}\times\mathbf{v}$ True / False

   (d) $\mathbf{u}\cdot\mathbf{u}=|\mathbf{u}|$ True / False

   (e) $\mathbf{u}\times\mathbf{u}=\mathbf{0}$ True / False

   (f) $(c+d)(\mathbf{u}+\mathbf{v})=c\mathbf{u}+d\mathbf{v}$ True / False

3. (a) Give a vector **v** perpendicular to the plane that contains the line $x = 1 + t$, $y = 2 + t$, $z = 3 - t$ and the line $x = -1 + 2t$, $y = 2$, $z = 1 + 2t$. **(3 points)**

$\mathbf{v} = \langle \quad , \quad , \quad \rangle$

(b) Find the angle θ between the planes $-2x + 4y + 2z = 12$ and $3x + y + z = -1$. **(4 points)**

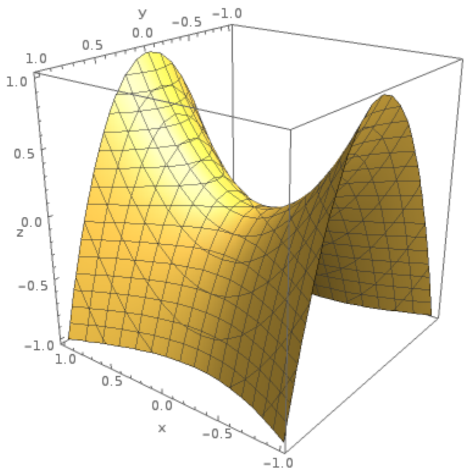
$\theta =$

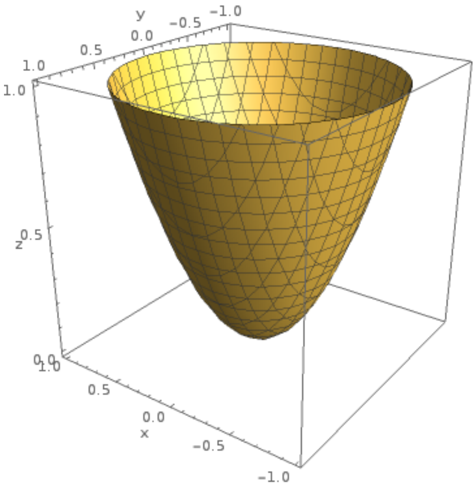
4. The plane P has normal vector $\langle 3,3,6\rangle$ and passes through $(0,-1,0)$. Find the shortest vector $\mathbf{v}$ from $(9,2,-3)$ to P. **(5 points)**

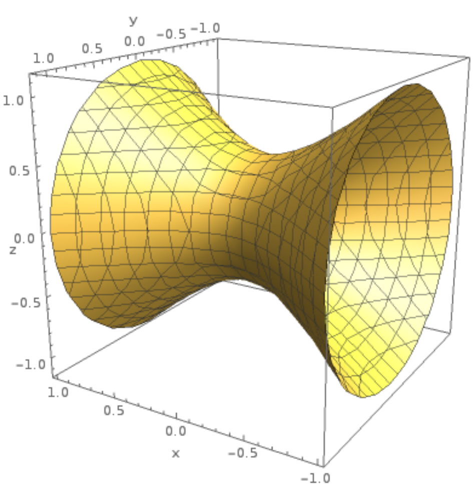
$\boxed{\mathbf{v} = \langle \quad , \quad , \quad \rangle}$

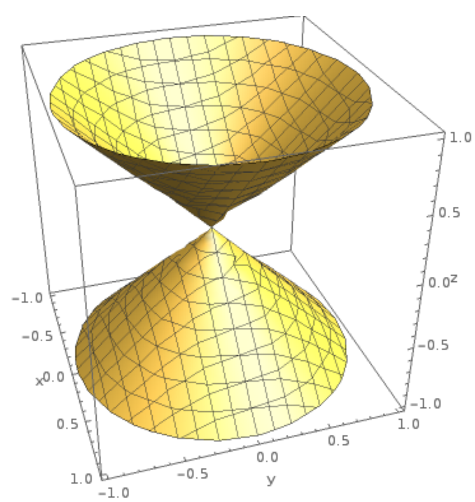
5. **(1 point each + 1 for at least three correct)** Identify the equations of each of the following graphs (write the letter of your selection below each graph):

(A) $x^2 + y^2 + z^2 = 1$	(E) $x^2 + y^2 - z^2 = 0$
(B) $x^2 + y^2 - z = 0$	(F) $x^2 - y^2 - z = 0$
(C) $x^2 - y^2 - z^2 = 1$	(G) $-x^2 + y^2 + z^2 = 1$
(D) $x^2 + y^2 + z^2 = 0$	(H) $x^2 + y^2 + z^2 = -1$

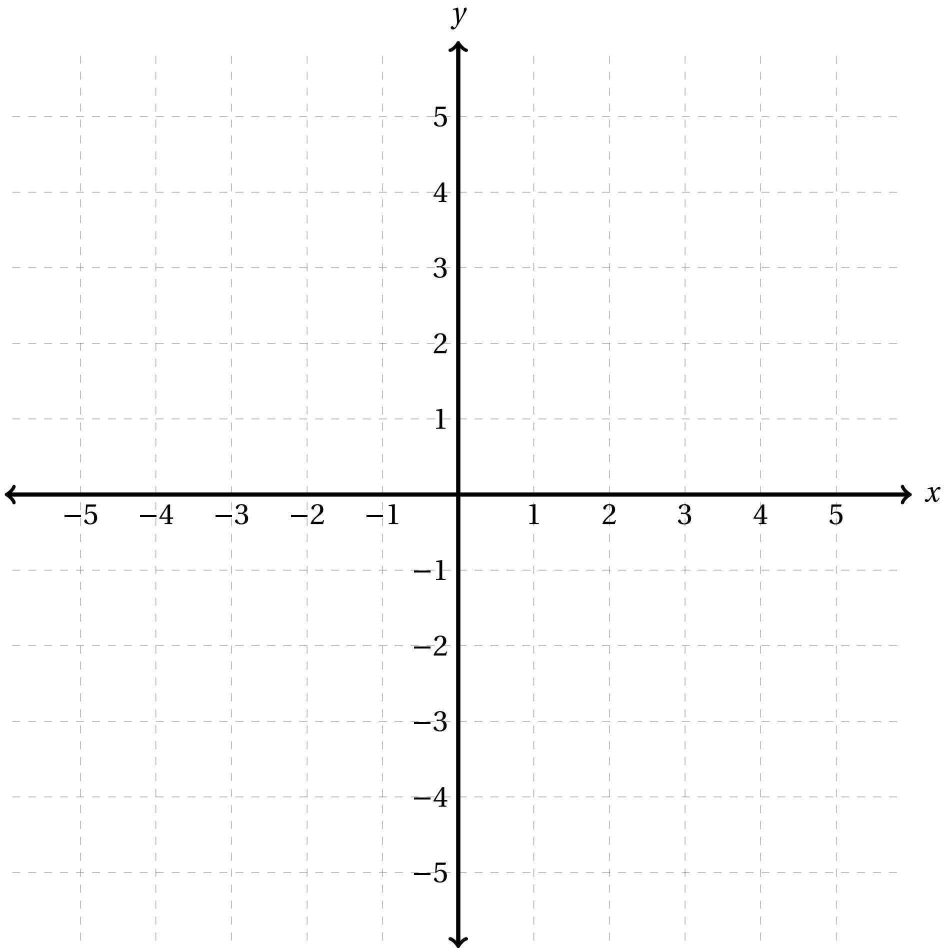








6. Sketch a contour map of $f(x, y) = x^2 - 4x + y^2 + 5$ for level curves corresponding to $z = 2, 5$ and 10. **(4 points)**



7. Consider each of the following limits. In each case does this limit exist (you must justify your answer)? If so, what is its value?

   (a) $\lim_{(x,y)\to(0,0)} y^4 + xy + 3$ **(1 point)**

   (b) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+2y^2}$ **(3 points)**

   (c) $\lim_{(x,y)\to(0,0)} \frac{3xy^2+x^2y}{x^2+y^2}$ **(3 points)**

8. **(2 points each)** Let $f(x, y) = x^3 + \sin(xy^2)$. Compute:

(a) $f_x = \boxed{}$

(b) $f_y = \boxed{}$

(c) $\dfrac{\partial^2 f}{\partial x \partial y} = \boxed{}$